Delay-Based Feedback Formation Control for Unmanned Aerial Vehicles with Feedforward Components

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Abstract. This paper deals with the delay-based feedback formation control problem with feedforward components for multiple unmanned aerial vehicle (UAV) systems. First, a moving equation of the leader-follower UAV formation system with regard to three directions is established, and the communication network topology for the agents is presented, where only position information is shared between each follower and the leader, while there is no communication among the followers. Second, by intensionally introducing time-delays into feedback control channel, a delay-based feedback formation control scheme with feedforward components is proposed for the UAV system. The sufficient conditions of asymptotical stability of close-loop system are derived, and the design method of the delayed formation controller is presented. The effectiveness of the delay-based feedback formation control scheme with feedforward components is verified based on simulation results, which shows than under the designed formation controller, the formation performance of the multiple UAV system can be guaranteed effectively.

Keywords: UAVs, formation control, leader-follower, time-delay, delay-based feedback

1 Introduction

Formation control of multi unmanned aerial vehicles (UAVs) plays an important role in the cooperative control of multi-agent system. Due to the characteristics of small size, light weight, and high flexibility, UAVs are widely used in many fields, such as volcano monitoring [1], target detection [2], coverage path planning [3], logistics delivery [4], large-scale rescue search [5], etc. Based on the single UAV, the number of carried equipments is small and the sensing range is limited. As a result, the single UAV can not meet the increasing real requirements. Therefore, the research issues and applications of UAV has developed from single platform to multi platforms. As one of the fundamental problems of multi UAV systems, the formation control problem of the UAVs have become more and more important in recent decades.

From the structure perspective, UAV formation mainly includes leader-follower method [6], behavior-based method [7], virtual structure method [8], artificial potential field method [9] and so on. Relatively, leader-follower-based formation is one of widely used schemes. For instance, Lai et al. presents the formation control method of four rotor UAV through distance feedback control [10]. Qiu et al. proposes a distributed close formation control method by the leadership in the pigeon group model [11]. In [12], the UAV formation is analyzed by decomposing the UAV systems into several simple ones in a three-dimensional space. In [13], a formation control scheme is developed, where the formation is realised via a guiding first and then following mode. A distributed optimal control method based on reinforcement learning for the trajectory tracking of heterogeneous UAV formation [14], a guidance law of trajectory tracking and cooperation under the directed communication topology is developed [15]. In [16], a new second-order nonlinear multi-agent distributed consistency algorithm is designed. In [17], a disturbance observer-based formation control scheme is presented for the UAV system.

As is known that time-delay is unavoidable phenomenon in real system due to the signal input and transmission. In [18], a formation tracking control problem of second-order multi-agent systems with time-varying delay is investigated. In [19], a model predictive formation controller is designed to reduce the effects of the time-delay on the UAV system. In [20], by considering the time-varying communication delay, a leader-following formation control of second-order nonlinear systems is studied. With regard to the recent progress of the formation control problem for the UAV systems, one can see [21–23], and the references therein. In general, time-delay plays negative effects on the system performance. However, for some real systems, proper time-delay can enhance the system performance. For example, in [24], time-delays are introduced to reduce the vibration of the offshore structures. Inspired by [24], in this paper, by artificially introducing time-delay into control channel, we aim to design a delay-based formation controller with feedforward component for the UAVs, and investigate the effects of the timed-delay on the formation performance of the UAVs. For a leader-follower UAVs, a communication network topology is presented first. Then, by introducing time-delays into feedback control channel, a delay-based feedback formation control scheme with feedforward components is proposed, the sufficient condition of asymptotical stability of system is derived, and the design method of the delayed formation controller is developed. Simulation results show that the delay-based feedback formation control scheme with feedforward components is effective to guarantee the formation performance of the multiple UAVs.

2 Problem formulation

In this section, a notion of network topology of unmanned aerial vehicles is presented, and a delayed feedback formation control problem of the unmanned aerial vehicles is formulated.

The communication topology among UAVs is describe by a directed graph. The index set of L followers is defined as $\mathcal{L} = \{1, 2, \cdots, L\}$. Let $\mathcal{G} = (\mathcal{L}_0, \varepsilon)$ donate a directed graph, where $\mathcal{L}_0 = \{0, \mathcal{L}\}$ donates an index set of the leader and L followers, and $\varepsilon \subseteq \mathcal{L}_0 \times \mathcal{L}_0$ is an edge set of paired unmanned aerial vehicles. The pairs of UAVs in the directed graph $\mathcal{G}$ are ordered. A directed path is a sequence of ordered edges (i, j), where $i, j \in \mathcal{L}_0$.

Suppose that the follower j $(j \in \mathcal{L})$ only can receive the position information sent from the leader $i = 0$. That is, the positions of the followers only depend on the leader's position P_0. Define an adjacency matrix as $A_c = [c_{ij}]$, where

$$c_{ij} = \begin{cases} 1, \ i = 0, \ j \in \mathcal{L} \\ 0, \ \text{others} \end{cases} \tag{1}$$

Denote the position and velocity of agent i by $p_i(t)$ and $v_i(t)$, respectively, $i = 0, 1, 2, \cdots, L$. Then one gets

$$\dot{p}_i(t) = v_i(t), \ i = 0, 1, 2, \cdots, L \tag{2}$$

For simplification purpose, the position and velocity information of each agent are further decomposed into three axes as X, Y, and Z as

$$p_i = \begin{bmatrix} p_{0x} & p_{0y} & p_{0z} \end{bmatrix}^T, \ v_i = \begin{bmatrix} v_{ix} & v_{iy} & v_{iz} \end{bmatrix}^T, \quad i = 0, 1, 2, \cdots, L \tag{3}$$

where p_0 and v_0 are predefined position and velocity of the leader.

Define

$$\hat{p}_i(t) = \hat{p}_0(t) - s_i(t), \ i \in \mathcal{L} \tag{4}$$

where $\hat{p}_i = [\hat{p}_{ix} \quad \hat{p}_{iy} \quad \hat{p}_{iz}]^T$ is the expected position of the agent $i, i = i = 0, 1, 2, \cdots, L$, and $s_i = [\hat{s}_{ix} \quad \hat{s}_{iy} \quad \hat{s}_{iz}]^T$ represents the relative distance between leader and follower i during the formation process, $i \in \mathcal{L}$.

By Newton's second law, one yields the motion equation of the agent i as

$$f_i(t) = ma_i(t) + kv_i(t) + \vartheta, \quad i \in \mathcal{L} \tag{5}$$

where $f_i = [f_{ix} \ \ f_{iy} \ \ f_{iz}]^T$, $a_i = \dot{v}_i$, k is the air damping coefficient, $\vartheta = [0 \ \ 0 \ \ mg]^T$ with m the mass of the UAV and g the acceleration of gravity.

Denote

$$\begin{cases} x_i = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & x_{i4} & x_{i5} & x_{i6} \end{bmatrix}^T \\ u_i = \begin{bmatrix} f_{ix} & f_{iy} & f_{iz} \end{bmatrix}^T \end{cases} \tag{6}$$

where

$$x_{i1} = p_{ix}, \ x_{i2} = v_{ix}, \ x_{i3} = p_{iy}, \ x_{i4} = v_{iy}, \ x_{i5} = p_{iz}, \ x_{i6} = v_{iz} \tag{7}$$

Then the state space model of follower i can be expressed as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dg,\ i \in \mathcal{L} \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k}{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{k}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\frac{k}{m} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{9}$$

Next, for follower i, we define the desired formation trajectory variable as:

$$x_i^s = \begin{bmatrix} x_{i1}^s & x_{i2}^s & x_{i3}^s & x_{i4}^s & x_{i5}^s & x_{i6}^s \end{bmatrix}^T,\ i \in \mathcal{L} \tag{10}$$

where

$$\begin{cases} x_{i1}^s(t) = \hat{p}_{0x}(t) - s_{ix}(t),\ x_{i2}^s(t) = \dot{\hat{p}}_{0x}(t) - \dot{s}_{ix}(t) \\ x_{i3}^s(t) = \hat{p}_{0y}(t) - s_{iy}(t),\ x_{i4}^s(t) = \dot{\hat{p}}_{0y}(t) - \dot{s}_{iy}(t) \\ x_{i5}^s(t) = \hat{p}_{0z}(t) - s_{iz}(t),\ x_{i6}^s(t) = \dot{\hat{p}}_{0z}(t) - \dot{s}_{iz}(t) \end{cases} \tag{11}$$

Then one gets

$$\dot{x}_i^s(t) = Ax_i^s(t) + Bq_i(t),\ i \in \mathcal{L} \tag{12}$$

where

$$q_i(t) = \begin{bmatrix} m[\ddot{\hat{p}}_{0x}(t) - \ddot{s}_{ix}(t)] + kx_{i2}^s(t) \\ m[\ddot{\hat{p}}_{0y}(t) - \ddot{s}_{iy}(t)] + kx_{i4}^s(t) \\ m[\ddot{\hat{p}}_{0z}(t) - \ddot{s}_{iz}(t)] + kx_{i6}^s(t) \end{bmatrix} \tag{13}$$

Define formation error vector as

$$e_i(t) = x_i^s(t) - x_i(t),\ i \in \mathcal{L} \tag{14}$$

and design a formation controller as

$$u_i(t) = u_{if}(t) + u_{ib}(t) \tag{15}$$

where u_{if} and u_{ib} are the feedforward and feedback control laws, respectively.

In this paper, we intend to design the delay-based feedback formation controller (15) for the UAV system (8) such that the formation error (14) satisfies:

$$\lim_{t \to \infty} e_i(t) = 0,\ i \in \mathcal{L} \tag{16}$$

To obtain the main results, the following Lemma is required.

Lemma 1. *[24] Let ζ be a differentiable function: $[\tau_1, \tau_2] \to R^n$, and $\tau = \tau_2 - \tau_1$. For any symmetric constant matrix $Z \in R^{n\times n} > 0$, and matrices $P_1 = [M_1 \quad M_2 \quad M_3]$ and $P_2 = [N_1 \quad N_2 \quad N_3]$ with $M_i, N_i \in R^{n\times n}, i = 1, 2, 3$, the following inequality holds:*

$$-\int_{\tau_1}^{\tau_2} \dot{\zeta}^T(s) Z \dot{\zeta}(s) ds \le \eta^T(t) \left(\Omega + \tau P_1^T Z^{-1} P_1 + \frac{\tau}{3} P_2^T Z^{-1} P_2 \right) \eta(t) \tag{17}$$

where

$$\eta(t) = \begin{bmatrix} \zeta^T(\tau_2) & \zeta^T(\tau_1) & \frac{1}{\tau} \int_{\tau_1}^{\tau_2} \zeta^T(s) ds \end{bmatrix}^T, \ \Omega = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ * & \phi_{22} & \phi_{23} \\ * & * & \phi_{33} \end{bmatrix} \tag{18}$$

and

$$\begin{cases} \phi_{11} = M_1 + M_1^T + N_1 + N_1^T, & \phi_{12} = -M_1^T + M_2 + N_1^T + N_2 \\ \phi_{13} = M_3 + N_3 - 2N_1^T, & \phi_{22} = -M_2 - M_2^T + N_2 + N_2^T \\ \phi_{23} = -M_3 - 2N_2^T + N_3, & \phi_{33} = -2N_3 - 2N_3^T \end{cases} \tag{19}$$

3 Design of the formation controller

In this section, the feedforward and feedback control components $u_{if}(t)$ and $u_{ib}(t)$ in (15) are designed, respectively. Specifically, the existence conditions of the feedback control component are derived.

To compensate the leader-related signal q_i and effects of gravity acceleration, design the feedforward controller as

$$u_{if}(t) = q_i(t) + Hg, \ i \in \mathcal{L} \tag{20}$$

where $H = [0 \ \ 0 \ \ m]^T$.

Remark 1. Note that the leader-related signal q_i (13) includes accelerations of the leader and desired positions and accelerations of followers, which are generalally determined by the topology of the formation problem. Therefore, the feedforward control component can be designed as (20) to compensate the dynamic offset between the leader and the followers thereby enhancing the formation performance of the AUV system (8).

From (8), (10), and (14) with (15) and (20), one gets

$$\dot{e}_i(t) = Ae_i(t) - Bu_{ib}(t), \ i \in \mathcal{L} \tag{21}$$

Design the feedback controller as

$$u_{ib}(t) = K_i e_i(t - d), \ i \in \mathcal{L} \tag{22}$$

where K_i is a 3×6 gain matrix of feedback controller to be designed, $d \ge 0$ is an artificial time-delay introduced.

Substituting (22) into (21) yields the closed-loop formation error system as

$$\dot{e}_i(t) = Ae_i(t) - BK_i e_i(t-d),\ i \in \mathcal{L} \tag{23}$$

The following Proposition provides the sufficient conditions of the asymptotical stability of closed-loop formation error system (23).

Proposition 1. *For given scalar $d \geq 0$, the formation error system* (23) *is asymptotical stable if there exist 6×6 matrices $X > 0$, $Y > 0$, $Z > 0$, $S >$, M_j, N_j, $j = 1, 2, 3$, and 3×6 matrices $K_i, i = 1, 2, \cdots, L$ such that*

$$\begin{bmatrix} \Lambda & \phi_{12} - XBK_i & \phi_{13} & A^T S & \sqrt{d}M_1^T & \sqrt{d}N_1^T \\ * & \phi_{22} - Y & \phi_{23} & -K_i^T B^T S & \sqrt{d}M_2^T & \sqrt{d}N_2^T \\ * & & \phi_{33} & 0 & \sqrt{d}M_3^T & \sqrt{d}N_3^T \\ * & & * & dZ - 2S & 0 & 0 \\ * & & * & * & -Z & 0 \\ * & & * & * & * & -3Z \end{bmatrix} < 0 \tag{24}$$

where $\Lambda = \phi_{11} + XA + A^T X + Y$.

Proof. Construct a Lyapunov-Krasovskii candidate functional as

$$\begin{aligned} V(e_i(t)) =& e_i^T(t)Xe_i(t) + \int_{t-d}^{t} e_i^T(s)Ye_i(s)ds \\ &+ \int_{-d}^{0} ds \int_{t+s}^{t} \dot{e}_i^T(\theta)Z\dot{e}_i(\theta)d\theta \end{aligned} \tag{25}$$

Taking the derivative of $V(e_i(t))$ with respect to t along the trajectory of (23) yields

$$\begin{aligned} \dot{V}(e_i(t)) =& e_i^T(t)\left(XA + A^T X + Y\right)e_i(t) - 2e_i^T(t)XBK_i e_i(t-d) \\ &- e_i^T(t-d)Ye_i(t-d) + d\dot{e}_i^T(t)Z\dot{e}_i(t) - \int_{t-d}^{t} e_i^T(s)Ze_i(s)ds \end{aligned} \tag{26}$$

Note that for any matrix $S > 0$, the following is true:

$$2\left[Ae_i(t) - BK_i e_i(t-d) - \dot{e}_i(t)\right]^T S\dot{e}_i(t) = 0 \tag{27}$$

Let

$$\alpha(t) = \begin{bmatrix} e_i(t) & e_i(t-d) & \frac{1}{d}\int_{t-d}^{t} e_i(s)ds & \dot{e}_i(t) \end{bmatrix}^T \tag{28}$$

Then, from (26) and (27), and by Lemma 1, one gets

$$\dot{V}(e_i(t)) = \alpha^T(t)[\chi + d\Pi_1^T Z^{-1}\Pi_1 + \frac{d}{3}\Pi_2^T Z^{-1}\Pi_2]\alpha(t) \tag{29}$$

where

$$\Pi_1 = \begin{bmatrix} M_1 & M_2 & M_3 & 0 \end{bmatrix}^T,\ \ \Pi_2 = \begin{bmatrix} N_1 & N_2 & N_3 & 0 \end{bmatrix}^T \tag{30}$$

and

$$\chi=\begin{bmatrix} \varLambda & \phi_{12}-XBK_i & \phi_{13} & A^TS \\ * & \phi_{22}-Y & \phi_{23} & -K_i^TB^TS \\ * & * & \phi_{33} & 0 \\ * & * & * & dZ-2S \end{bmatrix} \tag{31}$$

To guarantee the asymptotic stability of the error system (23), the following inequality is needed:

$$\chi+d\varPi_1^TZ^{-1}\varPi_1+\frac{d}{3}\varPi_2^TZ^{-1}\varPi_2<0 \tag{32}$$

which is equivalent to the one in (24) by Schur complements. This completes the proof.

To solve the gain matrix K_i in (22), multiply the left-hand side of the inequality (24) by diag$\{X^{-1},\ X^{-1},\ X^{-1},\ S^{-1},\ X^{-1},\ X^{-1}\}$ and its transpose, respectively, and denote $\bar{X}=X^{-1}$, $\bar{Y}=X^{-1}YX^{-1}$, $\bar{Z}=X^{-1}ZX^{-1}$, $\bar{S}=S^{-1}$, $\widetilde{Z}=S^{-1}ZS^{-1}$, $\bar{M}_j=X^{-1}M_jX^{-1}, \bar{N}_j=X^{-1}N_jX^{-1}, j=1,2,3$. Then one yields

$$\begin{bmatrix} \bar{\varLambda} & \bar{\phi}_{12}-B\bar{K}_i & \bar{\phi}_{13} & \bar{X}A^T & \sqrt{d}\bar{M}_1^T & \sqrt{d}\bar{N}_1^T \\ * & \bar{\phi}_{22}-\bar{Y} & \bar{\phi}_{23} & -\bar{K}_i^TB^T & \sqrt{d}\bar{M}_2^T & \sqrt{d}\bar{N}_2^T \\ * & * & \bar{\phi}_{33} & 0 & \sqrt{d}\bar{M}_3^T & \sqrt{d}\bar{N}_3^T \\ * & * & * & d\widetilde{Z}-2\bar{S} & 0 & 0 \\ * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & -3\bar{Z} \end{bmatrix}<0 \tag{33}$$

where $\bar{\varLambda}=\bar{\phi}_{11}+A\bar{X}+\bar{X}A^T+\bar{Y}$, and

$$\begin{cases} \bar{\phi}_{11}=\bar{M}_1+\bar{M}_1^T+\bar{N}_1+\bar{N}_1^T, & \bar{\phi}_{12}=-\bar{M}_1^T+\bar{M}_2+\bar{N}_1^T+\bar{N}_2 \\ \bar{\phi}_{13}=\bar{M}_3+\bar{N}_3-2\bar{N}_1^T, & \bar{\phi}_{22}=-\bar{M}_2-\bar{M}_2^T+\bar{N}_2+\bar{N}_2^T \\ \bar{\phi}_{23}=-\bar{M}_3-2\bar{N}_2^T+\bar{N}_3, & \bar{\phi}_{33}=-2\bar{N}_3-2\bar{N}_3^T \end{cases} \tag{34}$$

Based on above analysis, we have following Proposition.

Proposition 2. *For given scalar* $d\geq 0$*, if there exist* 6×6 *matrices* $\bar{X}>0$, $\bar{Y}>0$, $\bar{Z}>0$, $\widetilde{Z}>0$, $\bar{S}>0$, $\bar{M}_j$, $\bar{N}_j$, $j=1,2,3$, *and* 3×6 *matrices* $\bar{K}_i, i=1,2\cdots,L$ *such that the inequality* (33) *holds, then the gain matrices* $K_i, i=1,2,\cdots,L$ *of the delayed feedback controller* (22) *are solvable, and*

$$K_i=\bar{K}_i\bar{X}^{-1} \tag{35}$$

Remark 2. Proposition 2 provides a method to solve gain matrix K_i of feedback controller (22). In fact, for a given time-delay d artificially introduced, if the inequality (33) is feasible, then the gain matrix K_i can be computed. Further, combining with (20) and (22), the delayed feedback formation controller (15) can be obtained.

Remark 3. Based on the linear inequality (33), the maximum admissible time-delay $d_{\max}$ intentionally introduced can be computed for the UAV system (8), and the effects of different time-delay d on the formation performance of the UAV system are different, which will be disscussed below.

4 Simulation examples

In this section, two examples regarding two different formation patterns are presented to show the effectiveness of the proposed formation control schemes for a UAV system. Then the effects of the time-delays introduced on the formation performance of the system are discussed.

4.1 Parameters of the UAV system and formation paterens

In (8), suppose that there are eight followers, i.e., $L = 8$. The mass m of each agent is 5 kg, and the air dumping coefficient k is 3 N·s/m. The desired flight path $\hat{P}_0(t)$ of the leader is given by

$$\hat{P}_0(t) = [10t \;\; 10\sin(0.1t) \;\; 100(1 - e^{-0.1t})]^T, \; t \geq 0 \tag{36}$$

In the two cases of formation pattern, i.e., 1-shape and V-shape, the initial states and the desired offsets are listed as follows:

Case I. 1-shape formation pattern

$$\begin{aligned} x_i(0) &= \begin{cases} \begin{bmatrix} 0 & 0 & -10i & 0 & 0 & 0 \end{bmatrix}^T, i = 1, 2, 3, 4 \\ \begin{bmatrix} 0 & 0 & 10(i-4) & 0 & 0 & 0 \end{bmatrix}^T, i = 5, 6, 7, 8 \end{cases} \\ s_i(t) &= \begin{cases} \begin{bmatrix} 0 & 10i & 0 \end{bmatrix}^T, i = 1, 2, 3, 4 \\ \begin{bmatrix} 0 & -10(i-4) & 0 \end{bmatrix}^T, i = 5, 6, 7, 8 \end{cases} \end{aligned} \tag{37}$$

Case 2. V-shape formation pattern

$$\begin{aligned} x_i(0) &= \begin{cases} \begin{bmatrix} -80i & 0 & -20i & 0 & 0 & 0 \end{bmatrix}^T, i = 1, 2, 3, 4 \\ \begin{bmatrix} -80(i-4) & 0 & 20(i-4) & 0 & 0 & 0 \end{bmatrix}^T, i = 5, 6, 7, 8 \end{cases} \\ s_i(t) &= \begin{cases} (i(5 - \cos(0.1t) - 2\sin(0.1t) \begin{bmatrix} 8 & 3 & 0 \end{bmatrix}^T, i = 1, 2, 3, 4 \\ (i(5 - \cos(0.1t) - 2\sin(0.1t) \begin{bmatrix} 8 & -3 & 0 \end{bmatrix}^T, i = 5, 6, 7, 8 \end{cases} \end{aligned} \tag{38}$$

To investigate the performance of the UAV system under designed formation controller, we introduce two performance indices with respect to formation error and control cost as:

$$J_{ei} = \int_0^\infty e_i^T(t) e_i(t) dt, \quad J_{ui} = \int_0^\infty u_i^T(t) u_i(t) dt \tag{39}$$

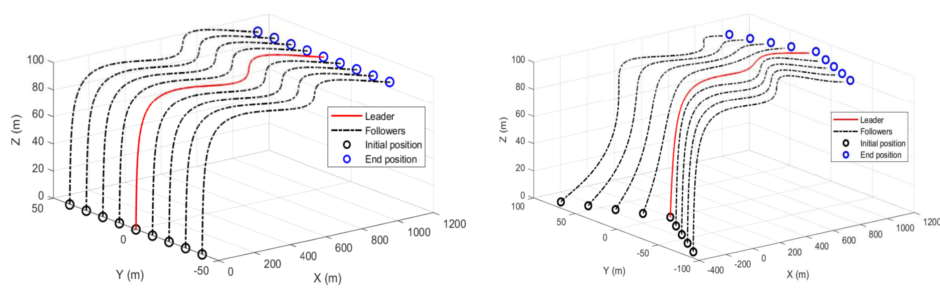
In what follows, in the aforesaid two cases, a delayed feedback formation controller with feedforward components is designed, and the performance of the formation control system and the effects of timed-delays on the formation control are discussed.

4.2 Formation control effects of the UAV

Based on (13) and (37) (or (38)), the feedforward component $u_{if}(t)$ in (20) can be determined. To design the feedback control component $u_{ib}(t)$, set $d = 0.35$ s. Then by Proposition 2, the gain matrix K_i is computed as

$$K_i = \begin{bmatrix} 2.4566 & 5.7279 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.4566 & 5.7279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.4566 & 5.7279 \end{bmatrix}, \ i = 1, 2, \cdots, 8 \quad (40)$$

Further the delay-based feedback formation controllers (DFFCs) in the form (15) can be obtained. The two formation controllers are denoted as DFFC1 for case 1 and DFFC2 for case 2, respectively. As the DFFC1 and DFFC2 are applied to the UAV system, the formation control results are depicted in Fig. 1(a) for case I and Fig. 1(b) for case II, respectively. The figures show that under the designed formation controllers, the followers can track the leader effectively. In addition, the 1-shape formation patter (case I) and V-shape formation pattern (case II) can be realised for all agents in the UAV system.



(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 1. Formation control result of UAV, $d = 0.35$ s.

4.3 Effects of time-delay on formation performance

By Proposition 2, it can be computed that the maximum admissible time-delay intentionally introduced is about 0.81 s. To analyze the effects of the time-delay on the formation performance and control cost by the UAV system, let the value of timed-delay d increase from 0s with a step 0.01s. Then under the DFFC1 and DFFC2 designed in subsection 4.2, one yields the performance indices (39) regarding formation errors and control cost of UAV, which are listed in Table 1 for case I and Table 2 for case II, respectively.

It is observed from Tables 1 and 2 that with the increase of timed-delay d, the whole formation error of the UAV system and the control cost become large gradually. Specifically, if $d = 0$s, the formation error and control cost are the smallest, while if $d = 1.33$ s, the former error and control cost are the largest, which can be found from Figs. 2(a) and 2(b) for $d = 0$s and Figs. 3(a)

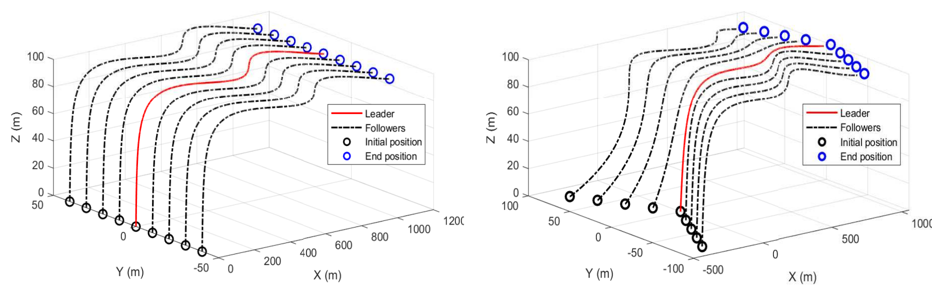
Table 1. Performance indices of UAV with DFFC1 for different time-delays.

d (s)	0	0.08	0.15	0.25	0.35	0.70	1.00	1.33
$J_e(10^3)$	1.4357	1.5152	1.5914	1.7134	1.8543	2.5959	3.9574	9.5016
$J_u(10^6)$	2.9689	2.9708	2.9727	2.9756	2.9789	2.9956	3.0248	3.1395

Table 2. Performance indices of UAV with DFFC2 for different time-delays.

d (s)	0	0.08	0.15	0.25	0.35	0.70	1.00	1.33
$J_e(10^5)$	3.3508	3.3928	3.4313	3.4900	3.5537	3.8406	4.2678	5.7346
$J_u(10^6)$	3.6681	3.7144	3.7583	3.8276	3.9066	4.3085	5.0181	7.8303

and 3(b) for $d = 1.33$s, respectively. In fact, if the value of introduced time-delay d is larger than 1 seconds, the relatively larger chattering phenomenon occurs. Consequently, the formation performance of the UAVs under the designed formation controller degrades gradually. Therefore, it is significant to choose a proper delay used for the formation controller design for the UAV system.

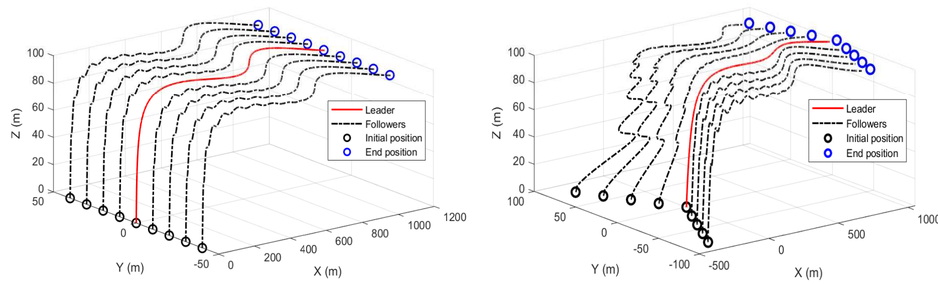


(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 2. Formation control result of UAV, $d = 0$ s.

5 Conclusion

In this paper, by introducing time-delays intentionally, a delay-based feedback formation control scheme with feedforward components has been developed. Based on leader-follower formation mode, a moving equation of UAVs has been established in a three-dimensional space. A delay-based feedback formation controller with feedforward components has been designed for the UAVs. By using Krasovskii stability theory, the existence and design method of the delayed formation controller have been obtained. Simulation results have been provided to demonstrate the effectiveness of the proposed formation control scheme for the multiple UAVs.



(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 3. Formation control result of UAV, $d = 1.33$ s.

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